

# Fuzzy Non-Instantaneous Deteriorating Inventory Model with Probabilistic Production, Demand and Time Dependent Deterioration Rate

K. Dhanam<sup>1</sup> and W. Jesintha<sup>2</sup>

<sup>1,2</sup>Department of Mathematics Government Arts College for Women (Autonomous)-Pudukkottai  
E-mail: <sup>1</sup>dhanamsakthivel@yahoo.co.in, <sup>2</sup>jacinthaw09@rediffmail.com

**Abstract**—In this paper, a fuzzy non-instantaneous deteriorating inventory model with probabilistic production, demand and time dependent deterioration rate have been proposed. The Lower-Upper (LU) bud fuzzy number is defined and its properties are given. The proposed model is formulated in fuzzy environment using LU-bud fuzzy number. ie., the parameters involved in this model are represented by LU-bud fuzzy number. The agreement index of LU-bud is explained and using this technique the total cost is defuzzified. Using the calculus technique optimal production quantity, deterioration rate are determined. A numerical example is given to illustrate both the proposed crisp model and fuzzy model.

**Keywords:** Probabilistic demand, LU-bud fuzzy number, agreement index technique.

AMS Subject Classification (2010) : 90B05.

## 1. INTRODUCTION

Inventory system is one of the main streams of the Operations Research which is essential in business enterprises and Industries. In business organization, inventory management is one of the major core competencies to compete in the global market place. Deterioration cannot be avoided in business scenarios. Deterioration is defined as change, damage, decay, spoilage, obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one product.

In reality, there are many situations where the demand rate depends on time. The demand of some items especially seasonable products like garments, shoes, mangoes, tomatoes etc., is low at the beginning of the season but increases as the season progresses. ie., changes with time.

A large number of inventory situations, the production and demand can be described by constant and deterministic inventory models. But, in real life and global market situations demand and various relevant costs are not exactly known. Randomness arises from the uncertainty in the production and demand capacity. In this situation, uncertainties are treated as randomness and are handled through probability theory is

addressed in [1,4&11]. Deterministic and Probabilistic models in inventory Control is given in [10].

A production inventory model for variable demand and production is explained in [5]. A continuous production control inventory model for deteriorating items is given in [6]. A Production Lot-size Inventory Model for Deteriorating Item is given in [8 & 12]. The production rate changes according to the number of working machines, production rates and demand rates change due to weather, economy, competition, seasonal promotion, customer status, and forecasting, etc. is discussed in [9 & 14]. The single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods and the essential characteristic of the models is that only a single time period and only a single procurement is given in [7]. Deterministic lot-size inventory model for deteriorating items with shortages and a declining market is discussed in [13]. Agreement index for a fuzzy number in introduction to fuzzy arithmetic is addressed in [2]. In some situations, the uncertainties are due to fuzziness. At first fuzzy concept in decision making used in [15]. Fuzzy goals, costs and constraints were introduced in [3]. The fuzzy linear programming model was formulated and an approach for solving linear programming model with fuzzy numbers has been presented in [16].

In an agricultural field during the cultivation, there is no deterioration, but during the harvesting, the rate of deterioration is linear and after completion of the harvesting, rate of deterioration is time dependent. Hence the non-instantaneous deteriorating inventory model is developed with probabilistic production, demand and time dependent deterioration rate. Probabilistic production follows increasing exponential distribution and probabilistic demand follows geometric distribution. The parameters are represented by LU-bud fuzzy number. This model is defuzzified by agreement index method. Finally a numerical example is provided to illustrate both the proposed crisp model and fuzzy model.

**2. ASSUMPTIONS AND NOTATIONS**

The following assumptions and notations are used throughout this paper :

**Assumptions**

1. The inventory system pertains single item.
2. The production process is done during the period (0,t<sub>2</sub>).
3. The production rate is a random variable, which follows increasing exponential distribution. pdf = λ e<sup>ap</sup>, λ > 0, a > 0.
4. The demand rate is a random variable, which follows geometric distribution.
5. The deterioration rate is time dependent.
6. During the period (t<sub>1</sub>,t<sub>3</sub>) the inventory level depleted due to time dependent deterioration only, during the period (t<sub>3</sub>,t<sub>4</sub>) the inventory level depleted due to demand and time dependent deterioration.

**Notations:**

- I<sub>1</sub>(t) -inventory level at any instant of time t, t<sub>1</sub> ≤ t ≤ t<sub>2</sub>;
- I<sub>2</sub>(t) -inventory level at any instant of time t, t<sub>3</sub> ≤ t ≤ t<sub>4</sub> ;
- I<sub>m</sub> -maximum inventory level at time t<sub>2</sub>;
- I<sub>s</sub> -maximum inventory level at time t<sub>3</sub>;
- p - production rate (random variable);
- d - deterioration rate (random variable);
- ̄s<sub>c</sub> -fuzzy setup cost per setup;
- ̄h<sub>c</sub> -fuzzy holding cost per unit per unit time;
- ̄d<sub>c</sub> -fuzzy deterioration cost per unit per unit time;

**3. MATHEMATICAL MODEL IN CRISP ENVIRONMENT:**

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost.

The rate of change of the inventory during the following periods are governed by the following differential equations

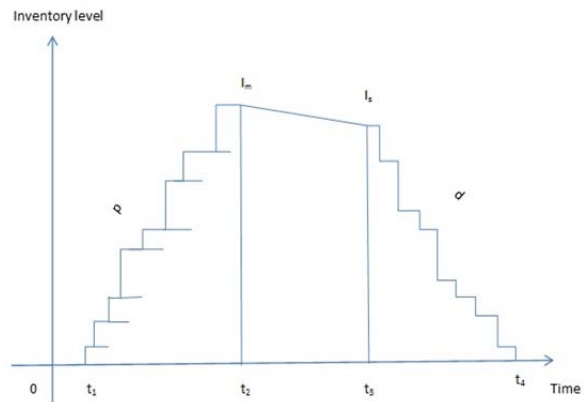
$$\frac{dI_1(t)}{dt} + (at+b)I_1(t) = -p, \quad t_1 \leq t \leq t_2 \text{----- (1)}$$

$$\frac{dI_2(t)}{dt} + (ct)I_2(t) = -d, \quad t_3 \leq t \leq t_4 \text{----- (2)}$$

with the boundary conditions I<sub>1</sub>(t<sub>1</sub>)=0, I<sub>1</sub>(t<sub>2</sub>)=I<sub>m</sub>, I<sub>2</sub>(t<sub>3</sub>)=I<sub>s</sub>, I<sub>2</sub>(t<sub>4</sub>)=0.

From equation (1),

From equation (1),



From equation (1),

$$I_1(t)e^{\int (at+b)dt} = -\int p e^{\int (at+b)dt} dt + c_1$$

$$I_1(t)e^{\frac{at^2}{2} + bt} = -p \left( \frac{e^{\frac{at^2}{2} + bt}}{at + b} \right) + c_1$$

using boundary condition I<sub>1</sub>(t<sub>1</sub>) = 0,

$$c_1 = p \left( \frac{e^{\frac{at_1^2}{2} + bt_1}}{at_1 + b} \right)$$

$$I_1(t) = p \left( \frac{e^{\frac{a(t_1^2 - t^2)}{2} + b(t_1 - t)}}{at_1 + b} \right) - \left( \frac{p}{at + b} \right)$$

using boundary condition I<sub>1</sub>(t<sub>2</sub>)= I<sub>m</sub>,

$$I_1(t_2) = p \left( \frac{e^{\frac{a(t_1^2 - t_2^2)}{2} + b(t_1 - t_2)}}{at_1 + b} \right) - \left( \frac{p}{at_2 + b} \right)$$

$$I_m = p \left( \frac{e^{\frac{a(t_1^2 - t_2^2)}{2} + b(t_1 - t_2)}}{at_1 + b} \right) - \left( \frac{p}{at_2 + b} \right)$$

From equation (2),

$$I_2(t)e^{\int (ct)dt} = -\int d e^{\int (ct)dt} dt + c_2$$

$$I_2(t)e^{\frac{ct^2}{2}} = -d \left( \frac{e^{\frac{ct^2}{2}}}{ct} \right) + c_2$$

using boundary condition I<sub>2</sub>(t<sub>3</sub>) = I<sub>s</sub>,

$$c_2 = \left( I_s + \frac{d}{ct_3} \right) e^{\frac{ct_3^2}{2}}$$

$$I_2(t) = \frac{-d}{ct} + \left( I_s + \frac{d}{ct_3} \right) e^{\frac{c(t_3^2 - t^2)}{2}}$$

using boundary condition  $I_2(t_4)=0$ ,

$$I_2(t_4) = \frac{-d}{ct_4} + \left( I_s + \frac{d}{ct_3} \right) e^{\frac{c(t_3^2-t_4^2)}{2}}$$

$$I_s = \left( \frac{d}{ct_4} e^{\frac{-c(t_3^2-t_4^2)}{2}} - \frac{d}{ct_3} \right)$$

Expected value of demand

$$\begin{aligned} E(d) &= \int_{t_3}^{t_4} df(d) dd \\ &= \int_{t_3}^{t_4} dp_1(1-p_1)^d dd \\ &= p_1 \left[ \frac{(1-p_1)^d}{\log(1-p_1)} \left( d - \frac{1}{\log(1-p_1)} \right) \right]_{t_3}^{t_4} \\ &= p_1 \left[ \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \left( t_3 - \frac{1}{\log(1-p_1)} \right) \right] \end{aligned}$$

Expected value of production

$$\begin{aligned} E(p) &= \int_{t_1}^{t_2} pf(p) dp \\ &= \int_{t_1}^{t_2} p(\lambda e^{kp}) dp \\ &= \lambda \left[ (p) \frac{e^{kp}}{k} - \frac{e^{kp}}{k^2} \right]_{t_1}^{t_2} \\ &= \lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right) \end{aligned}$$

Expected holding cost

$$\begin{aligned} E(HC) &= h_c \int_{t_1}^{t_4} (I_1(t) + I_2(t)) dt \\ &= h_c \int_{t_1}^{t_4} \left[ E(p) \left( \frac{e^{\frac{a(t_1^2-t^2)}{2} + b(t_1-t)}}{at_1+b} \right) - \left( \frac{E(p)}{at+b} \right) - \frac{E(d)}{ct} \right] dt \\ &\quad + \left( I_s + \frac{E(d)}{ct_3} \right) e^{\frac{c(t_3^2-t^2)}{2}} \\ &= h_c \left[ \left[ E(p) \left( \frac{-e^{\frac{a(t_1^2-t^2)}{2} + b(t_1-t)}}{(at_1+b)(at+b)} \right) - \frac{1}{a} \log(at+b) \right] + \right. \\ &\quad \left. E(d) \left( \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{ct_3(-ct)} - \frac{1}{c} \log t \right) + I_s \left( \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{-ct} \right) \right]_{t_1}^{t_4} \end{aligned}$$

$$\begin{aligned} &= h_c \left[ \frac{-E(p)}{(at_1+b)} \left( \frac{e^{\frac{a(t_1^2-t^2)}{2} + b(t_1-t)}}{(at_4+b)} - \frac{1}{(at_1+b)} - \frac{1}{a} \log \left( \frac{at_4+b}{at_1+b} \right) \right) \right] + \\ &E(d) \left( \frac{-e^{\frac{c(t_3^2-t^2)}{2}}}{c^2 t_3 t_4} + \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{c^2 t_3 t_1} \right) + I_s \left( \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{ct_1} - \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{ct_4} \right) \\ &\quad - \frac{1}{c} \log \left( \frac{t_4}{t_1} \right) \end{aligned}$$

setup cost =  $s_c$

Expected deteriorating cost

$$\begin{aligned} E(DC) &= d_c [(E(p) - I_m) + (I_m - I_s) + (I_s - E(d))] \\ &= d_c [(E(p) - E(d))] \end{aligned}$$

Expected total cost  $E(TC) = \text{Setup cost} + E(HC) + E(DC)$

Min  $E(TC)$

$$\begin{aligned} &= s_c + h_c \left[ \frac{-\lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right)}{(at_1+b)} \right. \\ &\quad \left( \frac{e^{\frac{a(t_1^2-t^2)}{2} + b(t_1-t)}}{(at_4+b)} - \frac{1}{(at_1+b)} - \frac{1}{a} \log \left( \frac{at_4+b}{at_1+b} \right) \right) \right. \\ &\quad + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \left( t_3 - \frac{1}{\log(1-p_1)} \right) \right) \\ &\quad \left. \left( \frac{-e^{\frac{c(t_3^2-t^2)}{2}}}{c^2 t_3 t_4} + \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{c^2 t_3 t_1} - \frac{1}{c} \log \left( \frac{t_4}{t_1} \right) \right) \right] \\ &\quad + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \left( t_3 - \frac{1}{\log(1-p_1)} \right) \right) \\ &\quad \left( \frac{-e^{\frac{-c(t_3^2-t^2)}{2}}}{ct_4} - \frac{1}{ct_3} \right) \left( \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{ct_1} - \frac{e^{\frac{c(t_3^2-t^2)}{2}}}{ct_4} \right) \\ &\quad + d_c \left( \lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right) - p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \right) \right. \\ &\quad \left. \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \right) \end{aligned}$$

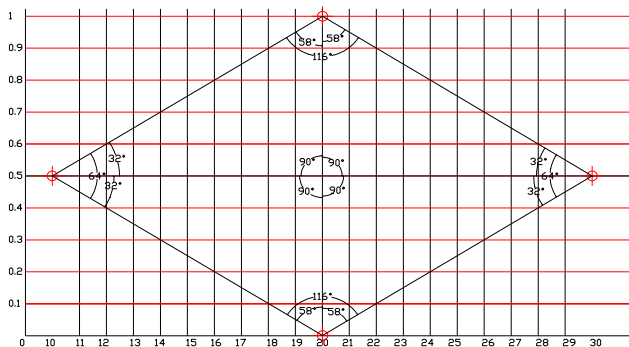
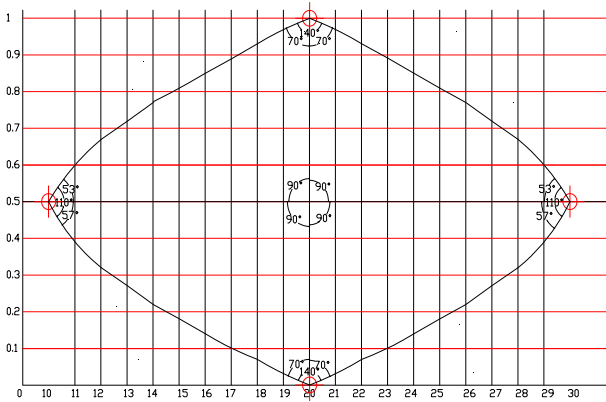
The expected total cost  $TC(t_1, t_2, t_3, t_4)$  is minimized by using the calculus technique. The optimal value of  $t_1, t_2, t_3$  and  $t_4$  have been obtained by solving the following equations.

$$\begin{aligned} \frac{\partial (ETC(t_1, t_2, t_3, t_4))}{\partial t_1} &= 0, \frac{\partial (ETC(t_1, t_2, t_3, t_4))}{\partial t_2} = 0, \\ \frac{\partial (ETC(t_1, t_2, t_3, t_4))}{\partial t_3} &= 0, \frac{\partial (ETC(t_1, t_2, t_3, t_4))}{\partial t_4} = 0 \\ \frac{\partial^2 (ETC(t_1, t_2, t_3, t_4))}{\partial t_1^2} &> 0, \frac{\partial^2 (ETC(t_1, t_2, t_3, t_4))}{\partial t_2^2} > 0, \\ \frac{\partial^2 (ETC(t_1, t_2, t_3, t_4))}{\partial t_3^2} &> 0, \frac{\partial^2 (ETC(t_1, t_2, t_3, t_4))}{\partial t_4^2} > 0, \end{aligned}$$

**4. LU-BUD FUZZY NUMBER AND ITS AGREEMENT INDEX METHOD: DEFINITION : (LU-BUD FUZZY NUMBER)**

A LU-bud fuzzy number  $\tilde{A}$  described as a normalized convex fuzzy subset on the real line  $R$  whose membership function  $\mu_{\tilde{A}}(x)$  is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0.5 & at \quad x = a, c \\ \frac{1}{2} \left\{ 1 - \left( \frac{x-a}{b-a} \right)^{\frac{n}{n-1}} \right\} & at \quad a \leq x \leq b \\ \frac{1}{2} \left\{ 1 - \left( \frac{c-x}{c-b} \right)^{\frac{n}{n-1}} \right\} & at \quad b \leq x \leq c \\ \frac{1}{2} \left\{ 1 + \left( \frac{x-a}{b-a} \right)^{\frac{n}{n-1}} \right\} & at \quad a \leq x \leq b \\ \frac{1}{2} \left\{ 1 + \left( \frac{c-x}{c-b} \right)^{\frac{n}{n-1}} \right\} & at \quad b \leq x \leq c \\ 0 \quad and \quad 1 & at \quad x = b \end{cases}$$



This type of fuzzy number be denoted as  $\tilde{A} = [a, b, c; W_A]$ , where  $W_A = 0.5$ , whose membership function  $\mu_{\tilde{A}}(x)$  satisfies the following conditions:

1.  $\mu_{\tilde{A}}$  is a continuous mapping from  $R$  to the closed interval  $[0,1]$ .
2.  $\mu_{\tilde{A}}$  is a convex function.
3.  $\mu_{\tilde{A}} = 0, 1$  at  $x = b$ .
4.  $\mu_{\tilde{A}} = 0.5$  at  $x = a$  &  $c$ .
5.  $\mu_{\tilde{A}}$  is strictly decreasing as well as increasing and continuous function on  $[a,b]$  &  $[b,c]$ .

**Properties:**

1. Opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at  $90^\circ$ .
3. In the horizontal and vertical diagonal, the base of the adjacent angles are equal.
4. The length of the horizontal diagonal, is twice of the length of the vertical diagonal.
5. If  $n=0$ , the LU-bud reduced to two lines, which intersect perpendicularly.
6. If  $n \geq 8$ , the LU-bud shape reduced to the diamond shape and the sum of the angles of the diagonal is  $360^\circ$ . (Figure:b)

**5. AGREEMENT INDEX METHOD :**

Let us define the concept of a fuzzy upper bound. A function  $h(x) ; x \in R$  such that

$$h(x) = \begin{cases} 1 & at \quad x \leq x_1 \\ \frac{x_2 - x}{x_2 - x_1} & at \quad x_1 \leq x \leq x_2 \\ 0 & at \quad x \geq x_2 \end{cases}$$

$h(x)$  represent by a fuzzy subset  $H \subset R$ . consider a fuzzy number  $A \subset R$ , which we call the agreement index of A with regard to H, the ratio being defined as

$$i(A, H) = \frac{\text{area of } A \cap H}{\text{area of } A} \in [0,1].$$

When **A is nonfuzzy**, that is,

$$\mu(A) = \begin{cases} 1, & x = n \\ 0, & x \neq n \end{cases}$$

then,  $i(A, H) = h(n)$ .

The agreement index of a LU-bud fuzzy number is of the form

When **H** is nonfuzzy, that is,

$$h(x) = \begin{cases} 1, & x \leq x_1 \\ 0, & x > x_1 \end{cases} \quad \text{in this case compute the area of A}$$

to the left of  $x_1$ .

$$i_G(A, H) = \frac{W_1}{(c-a)} [W_2 + aW_3 + bW_4 + cW_5 + W_6]$$

where,

$$W_1 = \frac{2n+1}{(n+1)}, W_2 = \left( \frac{(\alpha_2 - \alpha_1)(x_1 + x_2)}{2} \right),$$

$$W_3 = \left( \frac{n(2\alpha_2 - 1) \frac{2n+1}{n}}{2(2n+1)} - \alpha_2 \right),$$

$$W_4 = - \left( \frac{n(2\alpha_2 - 1) \frac{2n+1}{n}}{2(2n+1)} + \frac{n(1 - 2\alpha_1) \frac{2n+1}{n}}{2(2n+1)} \right),$$

$$W_5 = \left( \alpha_1 - \frac{n(1 - 2\alpha_1) \frac{2n+1}{n}}{2(2n+1)} \right), W_6 = \frac{-n}{2(n+1)}.$$

### 6. THE PROPOSED INVENTORY MODEL IN FUZZY ENVIRONMENT IS

$$\begin{aligned} \text{Min } E(TC) = \tilde{s}_c + \tilde{h}_c & \left[ \frac{-\lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right)}{(at_1 + b)} \right. \\ & \left( \frac{e^{\frac{a(t_1^2 - t_4^2)}{2} + b(t_1 - t_4)}}{(at_4 + b)} - \frac{1}{(at_1 + b)} - \frac{1}{a} \log \left( \frac{at_4 + b}{at_1 + b} \right) \right) \\ & + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \right. \\ & \left. \left( t_3 - \frac{1}{\log(1-p_1)} \right) \right) \left( \frac{-e^{\frac{c(t_3^2 - t_4^2)}{2}}}{c^2 t_3 t_4} + \frac{e^{\frac{c(t_2^2 - t_1^2)}{2}}}{c^2 t_2 t_1} - \frac{1}{c} \log \left( \frac{t_4}{t_1} \right) \right) \\ & + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \right. \\ & \left. \left( \frac{e^{\frac{-c(t_3^2 - t_4^2)}{2}}}{ct_4} - \frac{1}{ct_3} \right) \left( \frac{e^{\frac{c(t_3^2 - t_4^2)}{2}}}{ct_1} - \frac{e^{\frac{c(t_2^2 - t_1^2)}{2}}}{ct_4} \right) \right) \\ & + \tilde{d}_c \left( \lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right) - p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \right. \right. \\ & \left. \left. \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \right) \right) \end{aligned}$$

where ~ represents the fuzzification of the parameters and

$$\tilde{s} = (s_1, s_2, s_3); \tilde{h}_c = (h_1, h_2, h_3); \tilde{d}_c = (d_1, d_2, d_3) ;$$

Agreement index of the total cost is given by

$$\begin{aligned} \text{Min } i_G(E(TC), H) & = i_G(s_c, H) + i_G(h_c, H) \left[ \frac{-\lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right)}{(at_1 + b)} \right. \\ & \left( \frac{e^{\frac{a(t_1^2 - t_4^2)}{2} + b(t_1 - t_4)}}{(at_4 + b)} - \frac{1}{(at_1 + b)} - \frac{1}{a} \log \left( \frac{at_4 + b}{at_1 + b} \right) \right) + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) \right. \\ & \left. - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \left( t_3 - \frac{1}{\log(1-p_1)} \right) \right) \left( \frac{-e^{\frac{c(t_3^2 - t_4^2)}{2}}}{c^2 t_3 t_4} + \frac{e^{\frac{c(t_2^2 - t_1^2)}{2}}}{c^2 t_2 t_1} - \frac{1}{c} \log \left( \frac{t_4}{t_1} \right) \right) \\ & + p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \left( \frac{e^{\frac{-c(t_3^2 - t_4^2)}{2}}}{ct_4} - \frac{1}{ct_3} \right) \right) \\ & \left( \frac{e^{\frac{c(t_3^2 - t_4^2)}{2}}}{ct_1} - \frac{e^{\frac{c(t_2^2 - t_1^2)}{2}}}{ct_4} \right) + i_G(d_c, H) \left( \lambda \left( e^{kt_2} \left( \frac{t_2}{k} - \frac{1}{k^2} \right) + e^{kt_1} \left( \frac{t_1}{k} - \frac{1}{k^2} \right) \right) \right. \\ & \left. - p_1 \left( \frac{(1-p_1)^{t_4}}{\log(1-p_1)} \left( t_4 - \frac{1}{\log(1-p_1)} \right) - \frac{(1-p_1)^{t_3}}{\log(1-p_1)} \right) \right) \end{aligned}$$

The expected total cost  $i_G(E(TC(t_1, t_2, t_3, t_4)), H)$  is minimized by using the calculus technique. The optimal value of  $t_1, t_2, t_3$  and  $t_4$  have been obtained by solving the following equations.

### 7. NUMERICAL EXAMPLE

In one acre land, paddy is cultivated. Cultivation starts from sowing the seeds upto the harvesting, which takes the rate 19.5. After cultivation, the stocks are stored upto the cost is raised. After raising the cost, the supply starts in the rate of 100kg/day.

The following values of the parameter in proper unit were considered as input for the numerical analysis of the above problem,

$$\lambda_0 = 0.3; k = 19.5; a = 2; b = 5; c = 1; x_1 = 25; x_2 = 15; \alpha_1 = 0.815; \alpha_2 = 0.14; p = 0.7;$$

$$\begin{aligned} \tilde{s}_c &= [300 \quad 350 \quad 400]; \\ \tilde{h}_c &= [8 \quad 9 \quad 10]; \\ \tilde{d}_c &= [2 \quad 4 \quad 6]; \end{aligned}$$

Using the analytical expressions for ordering cost, holding cost, purchasing cost, shortage cost, the optimal values of  $t_1, t_2, t_3, t_4, I_m, I_s$  and deterioration rate are obtained and list out in the following tables for two cases.

Table

Model	$\tilde{s}_c$	$\tilde{h}_c$	$\tilde{d}_c$	t1	t2	t3
Crisp	300	8	2	0.1117	0.5989	0.7586
Fuzzy	350	8.5	4.5434	0.20	0.51	0.7065
Model	t4	Im	Is	deterioration rate	TC	
Crisp	1.4004	248.4	240.7	17.9537	14405	
Fuzzy	1.2640	273.9	267.4	19.0326	12267	

### Observation

From the above table, it should be noted that compare to crisp model the fuzzy model is very effective method. The expected total cost is obtained in fuzzy model is less than the crisp model.

### REFERENCES

- [1] Anderson, Sweeney and Williams "An Introduction to Management Science, quantitative approaches to decision making" seventh edition.
- [2] Arnold Kaufmann., Madan M and Gupta., (1991), "Introduction to fuzzy arithmetic theory and applications".
- [3] Bellman.R.E., & Zadeh.L.A.,(1970), "Decision-making in a fuzzy environment", Management Sciences, 17(4), 141-164.
- [4] T A Burley., G O'sullivan., "Operational Research".
- [5] Chao-Ton SU.,Chang-Wang LIN.,(1999),"A Production Inventory Model for variable demand and production" Yugoslav Journal or Operations Research 9, Number 2,197-206.
- [6] Chung KJ, Ting PS (1993) "A heuristic for replenishment for deteriorating items with a linear trend in demand", J Oper Res Soc 44:1235-1241.
- [7] Hamdy A Taha.,"Operations Research, An Introduction", fifth edition.
- [8] A. Kalam, D. Sama., S. K. Sahu & M. Mishra., "A Production Lot-size Inventory Model for Weibull Deteriorating Item with Quadratic Demand, Quadratic Production and Shortages", International Journal of Computer Science & Communication, Vol. 1, No. 1, January-June 2010, pp. 259-262.
- [9] Nirmal Kumar Mahapatra., Uttam Kumar Bera., Manoranjan Maiti.,(2012),"A Production Inventory Model with Shortages, Fuzzy Preparation Time and Variable Production and Demand", American Journal of Operations Research, 2, 183-192.
- [10] Ponnuru Ramalinga Karteek., Karri Jyoti.,(2014) "Deterministic and Probabilistic models in Inventory Control", IJEDR | Volume 2, Issue 3 | ISSN: 2321-9939.
- [11] Richard M. Feldman-Ciriaco Valdez-Flores,"Applied probability and stochastic processes", second edition.
- [12] G.P.Samanta.,Ajanta ROY.,(2004),"A Production inventory model with deteriorating items and shortages" Yugoslav Journal of Operations Research 14, Number 2, 219-230.
- [13] Wee HM.,(1995),"A deterministic lot-size inventory model for deteriorating items with shortages and a declining market " Comput Oper 22: 345-356.
- [14] Whitin TM.,(1957),"The theory of inventory management", second edition, Princeton University Press, Princeton.
- [15] Zadeh.L.A.,(1965),"Fuzzy sets. Information and control", 8, 338-353.
- [16] Zimmermann.H.J.,(1978),"Fuzzy linear programming with several objective function Fuzzy sets and systems",1,46-55.